Preliminary exam: Numerical Analysis, Part A, January 9, 2014

1. The system of linear equations $Ax = b$ is solved by an iterative method of the form, $x_{n+1} = Bx_n + c, n = 0,1,\ldots$. (a) Give necessary and sufficient condition for the method to converge $(x_n \to x, n \to \infty)$. (b) Present a proof for the conditions you give above. (c) If the matrix *A* is triangular, show that the Jacobi iterative method converges in a finite number of iterations.

2. Consider polynomial interpolation with *N* equidistant points $x_j = x_0 + jh,\ j = 1,2,..,N-1$ when the data $\{f(x_j)\}_{j=0}^{N-1}$ comes from a C^∞ function *f*. (a) Show that the interpolation error converges to 0 for fixed *N*, in an interval to be determined, as the distance between the points $h \to 0$. (b) Give an error estimate for the interpolation error and (c) give a proof for the error estimate.

3. A quadrature method has the form $\int_0^h f(x)dx \approx af(h/2) + bf(h)$. (a) Determine *a* and *b* such that the method is exact for the highest degree polynomial $p(x) = f(x)$ with the constraint $f(0) = p(0) = 0$. (b) Give an error estimate for the approximation if $f \in C^3, f(0) = 0$. (c) Describe how Richardson extrapolation can be used for an error estimate and to enhance the accuracy.