Numerical Analysis Prelim, Part II (Spring Material)

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1. Consider the following two-point boundary value problem:

$$y''(x) = f(y'(x), y(x), x), \quad 0 < x < 1$$
$$y(0) = \alpha, \ y(1) = \beta.$$

(a) Rewrite as first order system and approximate by the trapezoidal rule. What is the local truncation error?

(b) Determine an initial value method (shooting) for the problem on second order form with the mid-point rule as basic algorithm.

(c) Rewrite the problem on variational form when f is linear and determine a P_1 finite element algorithm for its approximation.

2. Given the following parabolic partial differential equation,

$$u_t(x,t) = a(x)u_{xx}(x,t) + b(x)u_x + f(x,t), \ 0 < x < 1, \ t > 0$$
$$u(x,0) = u_0(x), \ 0 < x < 1, \ u(0,t) = u(1,t) = 0, \ t > 0$$

(a) Devise a finite difference method that is based on forward difference in time (t) and centered differences in space (x) for uniform grids.

(b) Determine the order of the local truncation error and give conditions for regularity of the different functions in the equation that is required for this local truncation error.

(c) Use von Neumann analysis to determine L_2 stability of the approximation above if a and b are constants with $a > 0$.

3. (a) Derive a variational form of the partial differential equation below and specify the appropriate function spaces,

$$-\epsilon \Delta u + a \cdot \nabla u + u = f(x,y), \ x < 1, \ y < 1, \ a = (a_1, a_2),$$
$$u = g(x,y) \text{ at boundary.}$$

(b) Prove coercively for $\epsilon > 0$ and give error estimate for P_1 elements.

(c) When $\epsilon > 0$, what are appropriate boundary conditions? In this case, derive a discontinuous Galerkin method based upwind fluxes.