Preliminary exam: Numerical Analysis, Part A, (fall semester material)
RLM 9.166, 1:00-2:30 PM, August 20, 2019

Name ____ , EID ________________

1. (a) Define Cholesky factorization for the solution of strictly positive definite systems of linear equations. What is the computational cost for this factorization and that of LU factorization.

(b) Give the normal equation for the solution of the least squares problem $x = \underset{x}{\operatorname{argmin}} \|Ax - b\|_2^2$ where A is an m by n matrix. Discuss a role for Cholesky factorization in the solution of the normal equations. Will this approach always work? If it will, prove that, if not give counter example.

(c) The Moore-Penrose pseudo-inverse using singular value decomposition is another technique for solving the least squares problem. Describe this technique and give examples when this is advantageous over using the normal equations.

2. (a) Define Newton's method for solving a system of nonlinear equations $f(x) = 0$ and prove quadratic convergence in the scalar case under appropriate conditions on f and the starting value.

(b) In the Newton iteration for scalar problems $f'(x_j)$ plays an important role. Here x_j is the approximative solution at iteration step j. Prove convergence under appropriate conditions on f and the starting value if $f'(x_j)$ is replaced by $f'(x_0)$ throughout the iteration process.

(c) Describe two methods for solving scalar nonlinear equations, which do not require the derivative of f.

3. Consider the numerical integration approximating $I = \int_{-h}^{h} f(x)dx, h > 0$.

(a) Determine the numerical method of the highest order on the form (i.e. determine a and b),

$$I \approx af(0) + bf(1)$$

(b) Derive an asymptotic expansion in the step size parameter h for the method.
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(c) Give a brief description of the Romberg integration method.