Preliminary exam, Numerical Analysis, Part 1, algebra and approximation
12:30-2:30 PM, January 14, 2020

Please be on Zoom during the exam. Partial solutions give partial credit. Motivate your answers.

1. Consider the 2 by 2 linear system,

$$\begin{pmatrix} 2 & 4 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

(a) Formulate the Jacobi iterative method for the solution and investigate convergence properties.
(b) Can the Jacobi method be guaranteed to converge to the correct solution after, diagonal or upper triangular preconditioning?
(c) Will the power method to find the largest eigenvalue converge to the correct value? Can inverse iteration be used to find the largest eigenvalue?

2. Consider the 2 by 2 nonlinear system,

$$\begin{aligned} f(x, y) &= 0 \\ g(x, y) &= 0 \end{aligned}$$

(a) Formulate Newton's method for the solution and answer the question: is it enough to give conditions on *f* and *g* separately to guarantee convergence or do convergence also depend on the relation between these functions?
(b) Show quadratic convergence under appropriate conditions for the system above.
(c) Show how convergence of Newton's method for minimization can be derived from Newton's method for the solution of nonlinear systems. Does this include constrained minimization based on penalty methods.

3. Consider the numerical quadrature formula,

$$\int_0^h f(x)dx \approx af(0) + bf(c)$$

(a) What is the highest degree of polynomial that the formula is exact for if *a* and *b* are free to choose and *c* = *h*. Choose *a*, *b* optimally as function of *h*.
(b) What is the highest order of accuracy if also the best *c* is chosen and the function *f* is regular enough.
(c) What is the highest order of accuracy if *f* has a discontinuity in the interval (0, *h*)?