Algebraic Topology Prelim, January 2020

1. Let $K \subset S^3$ be the image of the standard inclusion of S^1 in S^3, i.e. the intersection of S^3 with $\mathbb{R}^2 \times \{(0,0)\} \subset \mathbb{R}^4$, and let T be the torus $S^1 \times S^1$. Let X be the space obtained from S^3 and T by identifying K with $S^1 \times \{point\}$ by some homeomorphism. Compute the singular homology of X.

2. Let S be a closed orientable surface of genus 2, and let X be the space obtained by attaching a 2-cell to S along the circle C shown.

(a) Compute $\pi_1(X)$.

(b) Show that for any $n = 1, 2, 3, ...$ there exists a connected regular covering space X_n of X with $\chi(X_n) = -n$.

(c) Explicitly describe two such covering spaces for $n = 2$, X_2 and Y_2 say, such that X_2 and Y_2 are not homeomorphic.

3. Let X be a connected CW-complex with $H_1(X) = 0$, and let T^n be the n-dimensional torus. Show that any map $X \to T^n$ is nullhomotopic.