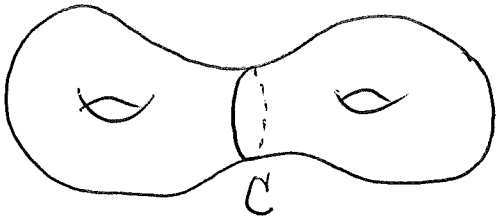
Algebraic Topology Prelim, January 2023

Do all three questions. The questions are weighted equally.

1. (a) Show that every non-surjective map from S^n to S^n has a fixed point.
(b) Show that (a) is false if S^n is replaced by T^n, $n \geq 2$.

2. Let S be a surface of genus 2, and let $C \subset S$ be the circle shown in the figure. Let X be the space obtained by gluing a Möbius band B to S by identifying the boundary of B with C by some homeomorphism.
(a) Compute the singular homology of X.
(b) Show that there is no retraction from X to the core circle of B.



3. Let X be the connected sum of three copies of $\mathbb{R}P^2$. Explicitly describe two connected 2-fold covering spaces $X_1 \to X$ and $X_2 \to X$, where X_1 is orientable and X_2 is non-orientable.