

Homework 10

Due Friday Apr 17

1. Let $f : (a, b) \rightarrow \mathbb{R}$ be twice differentiable at $x_0 \in (a, b)$. Show that if $f'(x_0) = 0$ and $f''(x_0) < 0$ then f has a local maximum at x_0 .
2. Compute $\int_0^1 x dx$ (the Riemann integral) directly from the definition (i.e. show that the integral exists and is equal to $\frac{1}{2}$).
3. (Ch 6, Question 1.) Suppose $x_0 \in [a, b]$, α is monotonic increasing on $[a, b]$ and continuous at x_0 , $f : [a, b] \rightarrow \mathbb{R}$ is given by $f(x_0) = 1$, $f(x) = 0$ for all other x . Prove that $f \in \mathcal{R}(\alpha)$ and that $\int f d\alpha = 0$.
4. (Ch 6, Question 2.) Suppose $f : [a, b] \rightarrow \mathbb{R}$ is continuous on $[a, b]$, $f \geq 0$ and $\int_a^b f(x) dx = 0$. Prove that $f(x) = 0$ for all $x \in [a, b]$.