

Homework 1

Due Friday Jan 30

1. Show that $\sqrt{3}$ is irrational.
2. Let $B = \{p \in \mathbb{Q} \mid p^2 > 3 \text{ and } p > 0\}$. Show that B has no smallest element.
3. (Ch 1, Question 3.) Prove Proposition 1.15.

For the following questions you may assume the usual properties of real numbers.

4. (Ch 1, Question 5.) Let A be a nonempty set of real numbers which is bounded below. Let $-A = \{-x \mid x \in A\}$. Prove that $\inf A = -\sup(-A)$.
5. Find the inf and sup of the following sets of real numbers, giving reasons if you can:
 - (a) $\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots\}$
 - (b) $\{\sqrt{2}, \sqrt{2 + \sqrt{2}}, \sqrt{2 + \sqrt{2 + \sqrt{2}}}, \dots\}$
6. (Ch 1, Question 8.) Prove that no order can be defined in the complex field that turns it into an ordered field. *Hint:* -1 is a square.