

## Homework 2

Due Friday Feb 6

- (Ch 1, Question 9.)
  - Suppose  $z, w \in \mathbb{C}$ ,  $z = a + bi$ ,  $w = c + di$ . Define  $z < w$  if  $a < c$  or if both  $a = c$  and  $b < d$ . Prove that this turns  $\mathbb{C}$  into an ordered set (not, of course, an ordered field).
  - Does this order have the least upper bound property? Prove or disprove.
- Suppose  $A$  and  $B$  are non-empty subsets of  $\mathbb{R}$ , both bounded above. Define  $(A + B) := \{x + y \mid x \in A, y \in B\}$ . Show that  $\sup(A + B) = \sup A + \sup B$ .
- (Ch 1, Question 15.) Under what conditions does equality hold in the Schwarz inequality? Justify your assertion if you can.
- Let  $f : X \rightarrow Y$  be a function, and let  $A, B \subset X$ . Prove that:
  - $f(A \cup B) = f(A) \cup f(B)$
  - $f(A \cap B) \subset f(A) \cap f(B)$
  - $f^{-1}(f(A)) \supset A$
- (Ch 2, Question 2.) A complex number  $z$  is said to be *algebraic* if there are integers  $a_0, \dots, a_n$ , not all zero, such that

$$a_n z^n + a_{n-1} z^{n-1} + \dots + a_1 z + a_0 = 0.$$

Prove that the set of algebraic numbers is countable. *Hint:* Show that for every  $N \in \mathbb{N}$  there are only finitely many equations with  $n + |a_0| + |a_1| + \dots + |a_n| = N$ . You may use the fact that a degree  $n$  polynomial has at most  $n$  roots.