Homework 3

Due Friday Feb 13

1. (Ch 2, Question 4.) Is the set of all irrational real numbers countable? Prove or disprove.

2. For $(x_1, x_2), (y_1, y_2) \in \mathbb{R}^2$ define:

   (a) ~~$d_1((x_1, x_2), (y_1, y_2)) := (x_1 - y_1)^2 + (x_2 - y_2)^2$~~

   (b) $d_2((x_1, x_2), (y_1, y_2)) := |x_1 - y_1| + |x_2 - y_2|$

   (c) $d_3((x_1, x_2), (y_1, y_2)) := \max\{|x_1 - y_1|, |x_2 - y_2|\}$

   (d) $d_4((x_1, x_2), (y_1, y_2)) := \begin{cases} \sqrt{x_1^2 + x_2^2} + \sqrt{y_1^2 + y_2^2} & \text{if} \quad (x_1, x_2) \neq (y_1, y_2) \\ 0 & \text{if} \quad (x_1, x_2) = (y_1, y_2) \end{cases}$

   Show that each of the above are metrics on $\mathbb{R}^2$. Illustrate by diagrams in the plane $\mathbb{E}^2$ what the neighbourhoods of these metric spaces are.

3. (Ch 2, Question 5.) Construct a bounded set of real numbers with exactly three limit points.

4. Show that the subset of $\mathbb{E}^2$ given by $\{(x, y) \in \mathbb{E}^2 \mid x < y\}$ is open.

5. (Ch 2, Question 9.) Let E° denote the set of all interior points of a set E (E° is called the <u>interior</u> of E).

   (a) Prove that E° is always open.

   (b) Prove that E is open if and only if $E^\circ = E$.

   (c) If $G \subset E$ and G is open, prove that $G \subset E^\circ$.

   (d) Prove that $(E^\circ)^c = \overline{(E^c)}$.

   (e) Do E and $\overline{E}$ always have the same interiors?

   (f) Do E and E° always have the same closures?