

Trigonometric Identities

Odd-even identities:

$$\begin{aligned}\sin(-x) &= -\sin x \\ \cos(-x) &= \cos x \\ \tan(-x) &= -\tan x\end{aligned}$$

Co-function identities:

$$\begin{aligned}\sin\left(\frac{\pi}{2} - x\right) &= \cos x \\ \cos\left(\frac{\pi}{2} - x\right) &= \sin x \\ \tan\left(\frac{\pi}{2} - x\right) &= \cot x\end{aligned}$$

Pythagorean identities:

$$\sin^2 x + \cos^2 x = 1$$

From this follow:

$$\begin{aligned}\sec^2 x &= 1 + \tan^2 x \\ \csc^2 x &= 1 + \cot^2 x\end{aligned}$$

Addition formulae:

$$\begin{aligned}\sin(A + B) &= \sin A \cos B + \sin B \cos A \\ \sin(A - B) &= \sin A \cos B - \sin B \cos A \\ \cos(A + B) &= \cos A \cos B - \sin A \sin B \\ \cos(A - B) &= \cos A \cos B + \sin A \sin B \\ \tan(A + B) &= \frac{\tan A + \tan B}{1 - \tan A \tan B} \\ \tan(A - B) &= \frac{\tan A - \tan B}{1 + \tan A \tan B}\end{aligned}$$

Many useful identities follow from the addition formulae:

Double-angle formulae: setting $A = B$

$$\begin{aligned}\sin 2\theta &= 2 \sin \theta \cos \theta \\ \cos 2\theta &= \cos^2 \theta - \sin^2 \theta \\ \cos 2\theta &= 2 \cos^2 \theta - 1 \\ \cos 2\theta &= 1 - 2 \sin^2 \theta\end{aligned}$$

Alternative very useful versions of the last two are:

$$\begin{aligned}\cos^2 x &= \frac{1}{2}(1 + \cos 2x) \\ \sin^2 x &= \frac{1}{2}(1 - \cos 2x)\end{aligned}$$

Product formulae:

$$\begin{aligned}\sin x \cos y &= \frac{1}{2}(\sin(x + y) + \sin(x - y)) \\ \cos x \sin y &= \frac{1}{2}(\sin(x + y) - \sin(x - y)) \\ \cos x \cos y &= \frac{1}{2}(\cos(x + y) + \cos(x - y)) \\ \sin x \sin y &= -\frac{1}{2}(\cos(x + y) - \cos(x - y))\end{aligned}$$