

SMALL EXPERIMENTS – OVERVIEW

For practical reasons, it is sometimes necessary to run experiments with just one observation per cell ($r = 1$).

Problems:

- Confidence intervals may be wide and power of hypothesis tests low.
- The estimate $msE = ssE/(n-v)$ for σ^2 in the Two-Way Complete model doesn't apply:

$$n-v = ab - ab = 0.$$

Possible remedies for the second problem:

1. If interaction is known in advance to be negligible, use Main Effects model.
2. Sometimes transformed data (especially logs) has no interaction.

e.g., if going from level 1 to level 2 of factor B *multiplies* response by 2.

3. *Method of Orthogonal Contrasts*: If certain contrasts are known in advance to be negligible, and if these contrasts are “orthogonal,” then the sums of squares of these contrasts can be pooled to obtain an estimate of error variance. (See Section 6.7.2.)

4. *Tukey's Test for Additivity* can be used if the size of the interaction term is expected to be proportional to the product of the main effects:

$$(\alpha\beta)_{ij} = \lambda\alpha_i\beta_j$$

(See Section 6.7.3, or Montgomery Section 5-3.7.)

5. Techniques applicable for “effect sparsity”: When number of treatment combinations is large and only a few contrasts are likely to be non-negligible. (See Section 7.5.)

Cautions:

1. With no replication, we can't expect to get much estimate of pure experimental error.
2. A method with error estimate biased *up* will give a *conservative* procedure:
 - Computed p-values will generally be larger than the true p-value.
 - Consequence: We fail to reject the null hypothesis some times when we would reject under an "exact" method.
 - Thus we might omit terms from models.
 - Methods that "pool" error from interactions deemed negligible are valid or conservative, *if* there is no data snooping.

3. A method with error estimate biased *down* will give a *liberal* procedure:
 - The computed p-value is generally smaller than the true p-value.
 - Consequence: a higher rate of falsely rejecting the null hypothesis.
 - Thus we might include unneeded terms in models.
 - Data snooping tends to have this effect.
4. Another method sometimes used is to use an "external estimate" of error.
 - This is risky: We can't be sure that an estimate obtained from a previous experiment really applies in a new experimental situation.