CONDITIONAL MEANS AND VARIANCES, PART II: MORE ON CONDITIONAL MEANS M 384G/374G

Summary of Part 1: The marginal mean of a random variable Y is the weighted average of the conditional means, when conditioned on a second random variable X. This says

$$E(Y) = \sum_{\text{all values } x \text{ of } X} \Pr(x) E(Y \mid X = x) \text{ (X discrete)}$$

or

$$E(Y) = \int_{-\infty}^{\infty} f_X(x) E(Y \mid x) dx \text{ (X continuous),}$$

where $f_X(x)$ is the probability density function (pdf) of X.

Note:

1. There are analogous results for conditioning on more than one variable.

2. The analogous result for *sample* means is

$$\bar{y} =$$

A second (related) relationship between marginal and conditional means for populations:

Consider E(Y|X) as a new random variable U as follows:

Randomly pick an x from the distribution of X.

The new r.v. U has value E(Y|X = x).

Example: Y = height, X = sex

Randomly pick a person from the population in question.

$$U = \begin{cases} E(Y \mid X = female) \text{ if the person is female} \\ E(Y \mid X = male) \text{ if the person is male} \end{cases}$$

Consider the expected value of this new random variable. (e.g., the expected value of the mean height for the sex of a randomly selected person from the given population. In this case, we would expect E(U) to depend on the proportion of the population which is of each sex.)

If U is discrete, then

$$E(U) = \sum_{\substack{All\ possible \\ values\ of\ U}} P(u)u$$

Example: For U = E(height | sex), the values taken on by U are

_______________ and ________________,

with respective probabilities ___________ and ______________,

so E(U) = ____,

which from Part I is just _______________________________ .

In other words,

E(E(ht|sex) =

The same reasoning works in general, showing that:

The expected value of the conditional means is the weighted average of the conditional means marginal mean, which from Part 1 is just the marginal mean
i.e.,

E(E(Y|X)) = weighted average of conditional means
= E(Y)