FACTORS AND INTERACTIONS

Example: Testing a new teaching method.

The question: Do students learn more under the experimental method than other the usual method, other things being equal?

Experimental design:

- Subjects randomly assigned to treatment and control groups.
- Post-test to measure outcomes.
- Pre-test to take into account prior learning
- Aptitude test to account for other initial differences.

Variables:

Response:

y = (post-test score) - (pretest score)

Predictors:

x_1 = score on aptitude test (a *covariate*)

$$x_2 = \begin{cases} 0 & \text{control group} \\ 1 & \text{experimental group} \end{cases}$$

Possible models:

Model I. $E(y \mid x_1, x_2) = \eta_0 + \eta_1 x_1 + \eta_2 x_2$

This says:

For the control group (x_2 = ___),

$E(y \mid x_1) =$

For the treatment group (x_2 = ___),

$E(y \mid x_1) =$

Possible Picture when $\eta_0 > 0$, $\eta_1 > 0$, $\eta_2 > 0$:

Exercise: Draw pictures for other cases of coefficients (e.g., $\eta_0 > 0$, $\eta_1 < 0$, $\eta_2 < 0$)

If Model I is correct, then

$\eta_2 > 0$ says:

$\eta_2 = 0$ says:

$\eta_2 < 0$ says:

Is this the correct model if, for example, the new method helps low aptitude students more than high aptitudestudents?

Model II. Adding an *interaction term* x_1x_2 to Model I gives:

$$E(y \mid x_1, x_2) = \eta_0 + \eta_1 x_1 + \eta_2 x_2 + \eta_3 x_1 x_2$$

This says:

For the control group (x_2 = ___),

$E(y \mid x_1) =$

For the treatment group (x_2 = ___),

$E(y \mid x_1) =$

If $\eta_2 > 0$ and $\eta_3 < 0$, we have the picture:

or:

or:

This says: The new method does ____________ for low aptitude students than for high aptitude students.

Exercise: Draw pictures and interpret the situation when $\eta_2 < 0$ and $\eta_3 > 0$.