INTRODUCTION TO SMOOTHING M 374G/384G

One aspect of regression is to see how the "center" of the conditional distributions varies as a function of the explanatory variable -- e.g., to express $E(Y|X = x)$ as a function of x.

A *smooth* is a curve constructed (by a computer algorithm) to go through or close to all points (x, f(x)) of certain function.

Examples:
- A "mean smooth" goes through or close to all points (x, $E(Y|X = x)$.
- A "median smooth" goes through or close to all points (x , $med(Y|X = x)$.

Example: In the fish data, we have seen both a median smooth (transparency) and a lowess mean smooth (constructed by arc).

Note: The median smooth was easy to construct for the fish data, since there were just a few values of the explanatory variable.

Example: Try this with the haystack data -- we need to choose the number of "slices," introducing the idea of a *smoothing parameter*.

Note: 1. What does the haystack smooth help us see in the data?
2. Arc also has a "slice smooth" function illustrating how a parameter in involved in creating a smooth.

The *lowess* (<u>lo</u>cally <u>we</u>ighted <u>sc</u>atterplot <u>s</u>moother) *smooth* can be found on most statistical software .

Outline of how the lowess curve is calculated
- Start with data points $(x_1, y_1), \ldots (x_n, y_n)$.
- Select a *smoothing parameter* f between 0 and 1. (We'll use f = 0.5 for illustration.)
- For each i,
  a. Look at the half (if f = ½; 1/4 if f = 1/4, etc.) of the data with x values closest to x_i.
  b. Fit a line (using weighted least squares -- we may talk about this later) to these points in a way that gives more weight to points with x closest to x_i.
  c. Replace y_i with y_i' = the y-value of the point on this line corresponding to x_i. (So y_i' "adjusts" y_i to be influenced by nearby data points.)
- After doing this separately for each i, repeat the procedure using points (x, y_i') (so the effect of points away from the trend will probably be less.)
- After a few iterations of this process, connect all the current "adjusted" points.