M365C (Rusin) HW1 – due Thursday, Sept 5 2019

1. Show that $A \subseteq B$ iff $B^c \subseteq A^c$

2. Is the relation "is a sibling of" an equivalence relation on the set of human beings? Discuss.

3. For any two natural numbers a and b we write $a|b$ to mean that there is a third natural number c with $b = ac$.

(a) Show $3|12$.

(b) Is $0|0$ correct?

(c) Find an infinite set S of natural numbers which is an ordered set using this relation.

4. Suppose $X \subseteq \mathbf{R}$ has an upper bound. Let $Y = \{-x \,|\, x \in X\}$. Show that Y has a lower bound. Then prove $\inf Y = -\sup X$.

5. Prove that between any two irrational numbers there exists another irrational number.

6. Let F be the set of real numbers which can be written as a polynomial in $a = 5^{1/3}$, e.g. $(7/11) + (1/2)a + 13a^4$. Is F a field? What about if $a = \pi$?

7. The *Well Ordering Principle* states that every non-empty set S of natural numbers has a smallest element. Prove this by induction. (Hint: let $P(n)$ be the statement, "If S contains the number n, then S has a least element.")

8. Let $S = \{x \,|\, x^3 > 7\}$. Show that if y is a rational number in S then there is a smaller rational number y' also in S. (Hint: Find y' using Newton's method. In order to show $y' \in S$ you may wish to write $y = a + h$ where a is the (real) cube root of 7.)