Math 408C (Rusin): Exam I, Sept 22 2011.
Please put your answers on the paper provided. You should show your work. Make sure you LABEL each solution with its problem number, and put your NAME on every sheet you wish to turn in.

1. A rectangular sheet of paper of area $A = 150$ cm^2 is rolled up to form a right circular cylinder by joining two opposite edges of the rectangle. Suppose this produces a cylinder having volume $V = 220$ cm^3; what is the radius of the cylinder?

2. (a) Sketch the graph of a function $f(x)$ which is defined for all real numbers x and which does *not* have an inverse. EXPLAIN why your function does not have an inverse.

(b) Sketch the graph of another function $g(x)$ which is defined for all real numbers x and which *does* have an inverse, $h(x)$; then sketch the graph of $h(x)$.

3. Find all solutions of the inequality

$$\ln(x^2 - 3x + 2) \leq 2\ln(x - 1),$$

expressing your answer in interval notation.

4. Suppose a function f is defined piecewise as follows:

$$f(x) = \begin{cases} 5 - 3x & \text{if} \quad x < -2 \\ 2x & \text{if} \quad -2 < x \leq 3 \\ x^2 - 3 & \text{if} \quad x > 3 \end{cases}$$

Determine the values of a for which

$$\lim_{x \to a} f(x)$$

exists. (Express your answer in interval notation.)

5. Find the value of b for which

$$\lim_{x \to 0} \left(\frac{1/(3x + b) - 2}{x} \right)$$

exists. Then, give the value of that limit.

6. What are all the vertical and horizontal asymptotes of the function

$$k(x) = \frac{x^3 - 2x^2 + x}{(x - 1)(x - 2)(x - 3)}$$

7. Determine the value of this limit, or explain why it does not exist.

$$\lim_{x \to 2} \left(\frac{1}{x - 2} - \frac{2}{x^2 - 2x} \right)$$

8. Determine whether

$$\lim_{x\to 0} x^2 \cos\left(\frac{1}{x}\right)$$

exists; if so, give its value.

9. For what values of c is the function

$$f(x) = \begin{cases} cx+7 & \text{if} \quad x \le 4 \\ 2x^2 - 3 & \text{if} \quad x > 4 \end{cases}$$

continuous at $x = 4$? Explain. (Your explanation should show that you understand what "continuous" means!)

10. If $\lim_{x\to 0} f(x) = +\infty$, what is

$$\lim_{u\to\infty} \frac{1}{u + f(1/u)}?$$

11. *Using the definition of the derivative,* compute

$$\frac{d}{dx}(2x + \frac{1}{x})$$

12. Observe that the graph of the following function passes through the origin:

$$f(x) = \begin{cases} 0 & \text{if} \quad x = 0 \\ x^3 \cos(\sqrt{x}) & \text{otherwise} \end{cases}$$

What is the line which is tangent to this graph at the origin?

13. A plutonium rod dropped from the top of the campus tower will fall to the ground quickly; t seconds after it is dropped, it will be approximately $h(t) = 256 - 16t^2$ feet off the ground.

(a) How long does it take the rod to fall to the ground?
(b) What is the rod's average speed during its fall?
(c) How fast is it falling at the moment of impact?

14. What is $r'(t)$ if

$$r(t) = \frac{2t-3}{4t+5}$$

15. It happens to be true that

$$\frac{d}{dx}\sin(x) = \cos(x) \qquad \text{and} \qquad \frac{d}{dx}\cos(x) = -\sin(x)$$

Use those pieces of information to compute the derivatives of these functions:

(a) $\tan(x)$
(b) $(\sin(x))^2 + (\cos(x))^2$

16. Find all horizontal lines which are tangent to the graph of $q(x) = x^3 - 6x + 5$.