Math 408C (Rusin): Exam II, Oct 25 2011. Put your NAME on each sheet you turn in.

1. Compute $h'(x)$ if $h(x) = \dfrac{x^2 - x + 2}{\sqrt{x}}$.

2. Find the points on the ellipse $x^2 + 2y^2 = 1$ where the tangent line has slope 2.

3. Here is a table showing the values of several functions at several points:

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	2	4	6
2	1	8	5	7
3	7	2	7	9

If $h(x) = f(g(x)) - \dfrac{f(x)}{g(x)}$, what is $h'(1)$?

4. Find all values of r for which $y = e^{rx}$ satisfies the equation

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = 0$$

5. If you pour salt out of a big container onto the table, it will form a cone; the more salt you pour, the larger the cone, but at all times you will see that the height of the cone equals the diameter of the cone's circular base. Suppose the salt is poured out at a rate of 30 cm^3/min. How rapidly is the height of the pile increasing, once the pile has risen to be 10 cm tall?

6. Compute $\displaystyle\lim_{x\to\infty} \frac{\sinh(x)}{\cosh(x)}$.

7. Compute the linearization (i.e. the Best Linear Approximation) of the function $Z(x) = \ln(1 + 3x)$ near $x = 0$. Then, *use* your linearization to estimate the value of $\ln(1.3)$

8. Use tools from Calculus to sketch the graph of $T(x) = \ln(x) - \frac{x}{100} + 50$. Your graph should make it clear where are the critical points, points of inflection, asymptotes, and intercepts with the coordinate axes. You do NOT need very accurate locations of these special points, but you should be able at least to tell how many there are. (If you wish to get numerical estimates for the coordinates of the points you will plot, you may find it helpful to know that $\ln(10) \approx 2.3$.)

9. Of all the points on the parabola $y = x^2$, which one is closest to the point $(0, 3)$? (Hint: you're trying to minimize a distance.)

10. (a) Explain why there is no value of x for which $\sec(x) = 0$.

(b) But $\sec(0) = 1$ and $\sec(\pi) = -1$. Doesn't the Mean Value Theorem tell us that a function that takes on positive and negative values will also equal zero somewhere? Do (a) and (b) (taken together) show that the Mean Value Theorem is incorrect? Explain.