BENNETT LINEAR ALGEBRA PRIZE EXAM Dec 12 2017

Name:_____ **UT EID:**_____

Linear Algebra Course:_____ **When?**_____ **Instructor:**_____

Permanent Mailing Address:_____

_____

E-mail address:_____

College (Natural Sciences, Engineering, etc.)_____

Submit your solutions on the sheets provided, with your name on each sheet. No calculators allowed. You must justify your claims.

---

1. Find the four real numbers a_i for which

$$\frac{8x+12}{(x^2-1)^2} = \frac{a_1}{x-1} + \frac{a_2}{(x-1)^2} + \frac{a_3}{x+1} + \frac{a_4}{(x+1)^2}$$

2. Suppose A and B are square matrices of the same size, and that $AB = 0$. Must $BA = 0$ too? (If you say "yes", give a proof; if you say "no", give a counterexample.)

3. Compute $\det(C)$ where C is the $n \times n$ matrix with $C_{ij} = 1$ if $i \neq j$ and $C_{ii} = 0$.

4. Suppose M is the 3×3 matrix which represents a 180° rotation around the line $x = y/2 = z/3$. (That's the line that contains the vector $\langle 1, 2, 3 \rangle$.) What are the eigenvalues of M? For extra credit give also the eigenvectors.

5. Suppose V is the vector space of all 3×3 matrices. Let $\mathcal{L}$ be the set of linear maps from V to V. This $\mathcal{L}$ is a vector space (you don't have to prove that).

(a) Show that for every invertible 3×3 matrix P, the function $f : V \to V$ given by $f(M) = PMP^{-1}$ is in $\mathcal{L}$ (i.e. show that f is a linear transformation).

(b) Are there other elements of $\mathcal{L}$ besides those in (a)? (If you say "no", give a proof. If you say "yes", find one.)

Answers will soon appear at `http://www.math.utexas.edu/users/rusin/Bennett/` .