ALBERT A. BENNETT CALCULUS PRIZE EXAM 12/6/09

Name:_____ **UT EID:**_____

Present Calculus Course:_____ **Instructor:**_____

Permanent Mailing Address:_____

_____

E-mail address:_____

School (Natural Sciences, Engineering, etc.)_____

Show all work in your solutions; turn in your solutions on the sheets provided. (Suggestion: Do preliminary work on scratch paper that you don't turn in; write up final solutions neatly and in order; write your name on all pages turned in.)

---

1. Find the sums of each of the following series. Simplify your answers.

(i) $\displaystyle\sum_{n=0}^{\infty}\left(\frac{x-1}{x}\right)^n$ where $x > 1$

(ii) $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{(\tan^{-1}(x))^{2n}}{(2n)!}$

2. Compute the following limits

(i) $\displaystyle\lim_{n\to\infty}\sum_{k=1}^{n}\frac{k^3}{n^4}\cos\left(\pi\frac{k^2}{n^2}\right)$

(ii) $\displaystyle\lim_{n\to\infty} n\int_2^{2+3\sin(1/n)} x^{-2}e^x\,dx$

3. Compute the indefinite integral

$$\int\frac{1}{1-x^{1/5}}\,dx$$

4. Find the volume of the solid torus (donut) obtained by rotating the unit disc $x^2+y^2\le 1$ about the line $x+y=6$.

5. There are 4 lines which are tangent to both of the circles $x^2+(y-3)^2=1$ and $x^2+(y+5)^2=4$. Find the equation of one of the lines.