ALBERT A. BENNETT CALCULUS PRIZE EXAM **Dec 4 2011**

Name:_____ **UT EID:**_____

Present Calculus Course:_____ **Instructor:**_____

Permanent Mailing Address:_____

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E-mail address:_____

School (Natural Sciences, Engineering, etc.)_____

Show all work in your solutions; turn in your solutions on the sheets provided. (Suggestion: Do preliminary work on scratch paper that you don't turn in; write up final solutions neatly and in order; write your name on all pages turned in.)

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1. Let $f(x) = e^{-x}\sin(x^3)/x$ and $g(x) = \ln(1+e^{-x})$. Compute

$$\lim_{x\to\infty} \frac{f(x)}{g(x)}.$$

2. For what values of a does this (improper) integral converge?

$$\int_a^\infty \frac{1}{\sqrt{|x^3(x-1)|}}dx$$

(*Possible Hint:* One approch uses the substitution $u = \frac{1}{x}$.)

3. Does the series

$$(\frac{-1}{1}) + (\frac{1}{2} + \frac{1}{3} - \frac{1}{4}) + (\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} - \frac{1}{9}) + \dots$$

converge? This series can also be written

$$\sum \frac{\varepsilon(n)}{n}, \quad \text{where} \quad \varepsilon(n) = \begin{cases} -1, & \text{if } n \text{ is a perfect square} \\ +1, & \text{otherwise} \end{cases}$$

4. Compute the first six terms of the Taylor series for $\sec(x)$, that is, determine the coefficients $a_0, \dots a_5$ in the expansion

$$\sec(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + \dots$$

5. Find the point on the paraboloid $z = 2x^2 + y^2$ which is closest to the plane $6x + 4y + z + 3 = 0$.