ALBERT A. BENNETT CALCULUS PRIZE EXAM **Dec 8 2013**

Name:_____ **UT EID:**_____

Present Calculus Course:_____ **Instructor:**_____

Permanent Mailing Address:_____

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E-mail address:_____

School (Natural Sciences, Engineering, etc.)_____

Show all work in your solutions; turn in your solutions on the sheets provided. (Suggestion: Do preliminary work on scratch paper that you don't turn in; write up final solutions neatly and in order; write your name on all pages turned in.)

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1. Evaluate the following limit (or explain why the limit does not exist):

$$\lim_{x\to 0} \frac{x \sin(\frac{1}{x})}{\ln(1+\sqrt{x})}$$

2. Which is larger — $\ln(2)$ or $\arctan(1)$? You must answer without a calculator of course, and memorized digits are also useless unless you can explain how those digits are computed. Use some calculus to describe these numbers.

3. Evaluate the following series, or explain why the series does not converge:

$$\frac{1}{1} + \frac{1}{(1+2)} + \frac{1}{(1+2+3)} + \ldots = \sum_{n=1}^{\infty} \frac{1}{\sum_{i=1}^{n} i}$$

4. Where does this function attain its maximum value?

$$F(x,y) = \int_x^{x+4} \int_y^{y+6} e^{-(u^2+t^2)}\, dtdu$$

5. Find all vectors v in $\mathbf{R}^3$ for which

$$v \cdot u_1 = 10, \qquad v \cdot u_2 = 11, \qquad v \cdot u_2 = 12,$$

where

$$u_1 = \langle 1,2,3 \rangle \qquad u_2 = \langle 4,5,6 \rangle \qquad u_3 = \langle 7,8,9 \rangle$$