ALBERT A. BENNETT CALCULUS PRIZE EXAM **Dec 10 2019**

Name:________________________ **UT EID:**________________________

Present Calculus Course:__________ **Instructor:**________________________

Permanent Mailing Address:____

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E-mail address:____

College (Natural Sciences, Engineering, etc.)____________________________

Submit your solutions *with all work shown* **on the sheets provided. No calculators allowed.** (Suggestion: Work first on scrap paper that you don't submit; write up final solutions neatly and in order, with your name on all pages submitted.)

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1. Find the positive number k for which the curves $y = \ln(x)$ and $y = x^k$ have exactly one point of intersection.

2. What is the minimum value of the sum of a convergent geometric series whose third term is 1 and whose other terms are all positive?

3. Evaluate $\displaystyle\int \frac{dx}{x^7 - x}$.

4. Compute $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 + 2y^2 - 5xy^2 - 6y^4}{x^2 + 2y^2 + 3x^2y + 4x^4}$ or show that the limit does not exist.

5. Evaluate the integral $\displaystyle\int_S f\, dA$ where $f(x, y) = 1/(1 - xy)$ and $S = [0, 1] \times [0, 1]$ is the unit square in the first quadrant. This is an improper integral; interpret this as

$$\lim_{t\to 1^-} \int_0^t \int_0^t \frac{1}{1 - xy}\, dx\, dy$$

(Hint: you will probably need either to find an alternative description of the integrand f or to transform the domain S with a change of variables.)

Answers will soon appear at `http://www.math.utexas.edu/users/rusin/Bennett/` .