ALBERT A. BENNETT CALCULUS PRIZE EXAM   Dec 07, 2021

Name:________________________ **UT EID:**____________________

Present Calculus Course:__________ **Instructor:**________________________

Permanent Mailing Address:____

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College (Natural Sciences, Engineering, etc.)________________________________

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1. Compute the value $f''(0)$ of the second derivative of f at $x = 0$, where

$$f(x) = \frac{(1+2x)^{1/2}(1+4x)^{1/4}(1+6x)^{1/6}\ldots(1+14x)^{1/14}}{(1+3x)^{1/3}(1+5x)^{1/5}(1+7x)^{1/7}\ldots(1+15x)^{1/15}}$$

2. Evaluate the limit:

$$\lim_{x\to 0^+}\left(\frac{1+2^x+3^x}{3}\right)^{1/x}$$

3. Evaluate the limit:

$$\lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{k^2}{n^3}+\frac{\sqrt{k}}{n^{3/2}}\right)$$

4. Compute the antiderivative:

$$\int\frac{d\theta}{5+2\cos(\theta)}$$

.

5. Let $f(x) = 1/(1+x+x^2)$ and let $\sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x + c_2 x^2 + \ldots$ be the Maclaurin series for f (i.e. the Taylor series of f around the origin). Compute $c_{36} - c_{37} + c_{38}$.

Answers will soon appear at `http://www.math.utexas.edu/users/rusin/Bennett/` .