ALBERT A. BENNETT CALCULUS PRIZE EXAM   5/5/07

Name:_____   UT EID:_____

Present Calculus Course:_____   Instructor:_____

Present Mailing Address:_____

_____

School (Nat'l Sciences, Engineering, etc.)_____

Show all your work in your solutions on the sheets provided. (Suggestion: Do preliminary work on scratch paper that you don't turn in; write up final solutions neatly and in order; write your name on all pages turned in.)

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1. David is slinging a rock on a string in a circle. The position of the rock at time t is given by $x = 3\cos(7t), y = 3\sin(7t), z = 6$. (David is 6 feet tall.) When $t = \pi/4$, David releases the string, and the rock flies in a straight line (ignore gravity) toward Goliath. Find parametric equations for the line.

2. A car is in Austin at 10:00 a.m. It travels 100 miles along I-35 and arrives in Waco at 12:00 noon. Show that, at some time between 10:00 and 12:00, the car was traveling exacvtly 50 miles per hour.

3. Each of the following is the Maclaurin Series (Taylor Series centered at 0) for some function. In each case find the function.

   (i) $1 + x + x^2 + x^3 + \ldots + x^n + \ldots$

   (ii) $1 + 2x + 3x^2 + 4x^3 + \ldots + nx^{n-1} + \ldots$

   (iii) $2 \cdot 1 + 3 \cdot 2x + 4 \cdot 3x^2 + 5 \cdot 4x^3 + \ldots + n \cdot (n-1)x^{n-2} + \ldots$

   (iv) $1 + 3x^2/1! + 3^2x^4/2! + 3^3x^6/3! + \ldots + 3^nx^{2n}/n! + \ldots$

4. Compute the indefinite integral $\int ((2x + 2)/(x^3 + 2x^2 + 2x))\, dx$.

5. There are two lines in the xy-plane that pass through the origin and are tangent to the curve $y = x^5 + x + 5$. Find the slopes of the two lines.