## PRELIMINARY EXAMINATION IN TOPOLOGY: PART 2

January 2023 2 Hours

Work all 3 problems. Explain your work carefully. The problems are weighted equally.

- 1. State clearly whether each of the following unrelated assertions is true or false. *Prove* your answer with a deductive argument, an example, a counterexample, etc.
  - (a) Let  $\Sigma$  be a smooth connected oriented 2-manifold with Euler number -2. Then there exists a degree 1 smooth map  $S^1 \times \Sigma \to S^3$ .
  - (b) Let  $L \subset \mathbb{A}^3$  be an affine line in real affine 3-space. If  $f: \mathbb{R} \to \mathbb{A}^3$  is transverse to L, then  $f(\mathbb{R}) \cap L = \emptyset$ .
  - (c) Suppose  $f: \mathbb{RP}^3 \to \mathbb{RP}^3$  is smooth. Then the Lefschetz number L(f) is odd.
- 2. Let C denote the subset of the real affine plane  $\mathbb{A}^2$  with coordinates x, y that is cut out by the equation

$$\frac{x^2}{9} + \frac{y^2}{4} = 1.$$

- (a) Draw a picture of C and display the orientation as the boundary of the compact region C bounds.
- (b) Compute

$$\int_C x \, dy.$$

Use the orientation of C in part (a).

- 3. Let Y be a smooth manifold and suppose  $Z \subset Y$  is a closed submanifold. For each of the following pairs  $Z \subset Y$ , either (i) produce a compact manifold X and a smooth map  $f: X \to Y$  such that the mod 2 intersection number  $\#_2(f, Z) = 1$ , or (ii) prove that no such X and f exist.
  - (a)  $S^1 \subset S^3$ , where  $S^1$  is equatorial
  - (b)  $\mathbb{RP}^1 \subset \mathbb{RP}^3$ , the image of (a) under the quotient by the antipodal map
  - (c)  $S^2 \times \text{pt} \subset S^2 \times S^2$
  - (d) pt  $\times \mathbb{RP}^2 \subset \mathbb{RP}^2 \times \mathbb{RP}^3$ , where  $\mathbb{RP}^2 \subset \mathbb{RP}^3$  is the projectivization of a subspace  $\mathbb{R}^3 \subset \mathbb{R}^4$