**Preliminary exam, Numerical Analysis, Part I, algebra and approximation**
**9:00 AM – 11:00 PM, January 10, 2024**

1. (a) Show how singular value decomposition (SVD) can be used to solve overdetermined systems of linear equations or, so called, least squares problems.
(b) Underdetermined systems of linear equations typically do not have unique solutions. What characterizes the solution when SVD in the form of the Moore-Penrose inverse are used to produce a solution. Motivate your answer.
(c) Householder transformations can be used to similarity transform a general square matrix to upper Hessenberg form. Describe how this is done and explain the difference to using Householder transformations in connection to SVD with transformation to bidiagonal matrices.

2. (a) Formulate Newton's method for a system of nonlinear equations.
(b) Give a geometric interpretation of Newton's method for solving scalar nonlinear equations.
(c) Prove that Newton's method converges for solving scalar nonlinear equations even if it is modified such that the term $f'(x_n)$ is fixed $(= f'(\bar{x}))$ under suitable assumptions. One assumption would, for example, be that $x_0$ and $\bar{x}$ are close to the true solution.

3. (a) Show how linear and piecewise linear Lagrange interpolation can be used to generate the trapezoidal and composite trapezoidal rule respectively for numerical integration.
(b) Use the error estimate in Lagrange interpolation to give an error estimate for the trapezoidal rule for numerical integration.
(c) Prove that linear B-spline interpolation (the same as piecewise linear Lagrange interpolation) is local in the sense that a change of one interpolation value only affects the interpolant on a domain of local support. Also prove that this is not true for quadratic B-splines over the real line.