

PRELIMINARY EXAMINATION IN ANALYSIS

Part II, Complex Analysis

August 11, 2022

The four problems carry equal weights. Full credit requires a complete sequence of arguments that can be understood by a typical student at this level. Partial credit is given for useful approaches and relevant partial results.

- 1.** Determine all analytic functions f on the strip $|\operatorname{Im} z| < 2$ that satisfy $|f(z)| = |z + 1|$ on the circle $|z| = 1$, have simple zeros at $\pm i/2$ and no other zeros in the disk $|z| < 1$.
- 2.** Describe the set of all functions $f : \mathbb{C} \setminus \{0\} \rightarrow \mathbb{C}$ that are analytic and one-to-one.
- 3.** Determine the partial fraction expansion for the function $z \mapsto \frac{1}{z \cos \sqrt{z}}$.
- 4.** Let D be an open subset of \mathbb{C} and R a closed proper subset of \mathbb{C} , both non-empty. Let $n \mapsto f_n$ be a sequence of analytic functions on D that take values in R . If $f_n \rightarrow f$ pointwise on D , show that f is analytic on D .