

Algebraic Topology Prelim, January 2021

1. Let S^2 be the unit sphere in \mathbb{R}^3 , D^2 the unit disk in the (x, y) -plane, and J the interval $\{(0, 0, z) : -1 \leq z \leq 1\}$. Let $X = S^2 \cup D^2 \cup J$.
 - (a) Compute the singular homology groups $H_i(X)$, $i \geq 0$.
 - (b) Exhibit a connected 2-fold covering space of X .

2. Let F be a compact, connected surface (orientable or non-orientable), with a single boundary component C . Show that there is no retraction from F onto C .

3. (a) Show that any map $S^2 \rightarrow T^2$ is null-homotopic.
(b) Show that there is a map $T^2 \rightarrow S^2$ that is not null-homotopic.