THE UNIVERSITY OF TEXAS AT AUSTIN
DEPARTMENT OF MATHEMATICS

The Preliminary Examination in Probability
Part I

Thu, Aug 19, 2021

Problem 1. Let X_n be a sequence of random variables taking values in $\mathbb{N}$. Is it true that

$$X_n \text{ converges a.s. if and only if } X_n \text{ converges in probability}$$

? If it is, give a proof. Otherwise, give a counterexample.

Problem 2. Let $X_1, X_2, \dots$ be i.i.d random variables with values in $\mathbb{Z}^2$, where X_1 is uniformly distributed in $\{(k,m) : k \in \{-1,0,1\}, m \in \{-1,0,1\}\}$ (9 possible values, each happens with probability 1/9). Let $S_n = \sum_{i=1}^{n} X_i \in \mathbb{Z}^2$. Show that $\frac{S_n}{\sqrt{n}} \xrightarrow{d} S^*$, and find the distribution of S^*.

Problem 3. Give an example of a submartingale $\{X_n\}_{n\in\mathbb{N}_0}$ with the property that $X_n \to -\infty$, a.s., but $\mathbb{E}[X_n] \to +\infty$, as $n \to \infty$.