PRELIMINARY EXAMINATION IN TOPOLOGY: PART 1

August 2021   2 Hours

Work all 3 problems. Explain your work carefully.

1. Let
$$X = (S^1 \times S^1) - (2 \text{ distinct points})$$

be a torus with 2 points removed and

$$Y = \mathbb{RP}^2 - (3 \text{ distinct points})$$

the real projective plane with 3 points removed. Prove or disprove: X and Y are homotopy equivalent.

2. Prove that $(S^a \times S^b) - \text{pt}$ and $(S^c \times S^d) - \text{pt}$ are homeomorphic if and only if $\{a, b\} = \{c, d\}$.

3. Let X be got from the disjoint union of $[0, 1]$ and two copies of $\mathbb{RP}^2$ by identifying 0 with a point of one copy of $\mathbb{RP}^2$ and 1 with a point of the other.

   (a) Describe all connected covering spaces $Y \to X$ (up to the topological type of Y).

   (b) Which of the covering spaces in (a) is regular?

   (c) Prove that X is not the total space of a nontrivial covering space.