Preliminary exam, Numerical Analysis, Part 1, algebra and approximation
1:00-2:30 PM, August 18,2020

1. (a) Define Householder transformations and show how they can be used in to transform matrix to upper Hessenberg form as part of a QR algorithm.
(b) Continue and describe the QR algorithm for computing eigenvalues and show that the algorithm results in a similarity transformation.
(c) Show how a QR decomposition can be used to compute the least square solution of a overdetermined systems of linear equations.

2. (a) Define Newton's method for the minimization of a function $f(x), x \subset R^d$, which has a unique minimum.
(b) Prove convergence for *d* = 1 under suitable conditions.
(c) If the minimization is constrained by linear constraints $Bx - b \leq 0$ where *B* is a matrix, show how a method for unconstrained minimization as, for example, Newton's method can be applied by adding a penalty function to $f(x)$.

3. The midpoint rule for numerical quadrature is,

$$\int_{-h}^{h} f(x)dx \approx 2hf(0).$$

(a) Determine an error estimate for the approximation.
(b) Derive an asymptotic expansion in the parameter *h* for the composite midpoint rule.
(c) Show how an asymptotic error expansion can be used in Richardson extrapolation to improve the accuracy of a method like the composite mid point rule.