Preliminary exam: Numerical Analysis, Part B, (spring semester material)
RLM 9.166, 2:40-4:10, August 20, 2019

Name:____ EID:____________________

1. Given the ordinary differential equation initial value problem,

$$x'(t) = f(x(t)), \quad t > 0,$$
$$x(0) = x_0$$

and the related two step linear multistep method

$$x_{n+2} - x_n = h(af(x_{n+2}) + (2-a)f(x_{n+1})$$

(a) For which values of a is the method Dahlquist (zero) stable
(b) For which values of a is the method of A-stable?
(c) Determine the order of accuracy as function of *a,* (assuming $x_1 = x(h)$).

2. Given the following parabolic PDE,

$$u_t = \nabla \cdot a(x,y)\nabla u - cu + f(x.y), t > 0, 0 < x < 1, 0 < y < 1,$$
$$0 < a \leq a(x,y) \leq A, c > 0,$$
$$u = 0, x = 0 \; and \; x = 1, 0 < y < 1,$$
$$u_y = \theta u + g(x), y = 0 \; and \; y = 1, 0 < x < 1,$$
$$u(x,y,0) = u_0(x,y), 0 < x < 1, 0 < y < 1,$$

(a) Rewrite the equation on weak form and discuss relevant function spaces
(b) Describe briefly a finite element method for this PDE
(b) Show that the bilinear form, which is relevant for right hand side in this equation as an elliptic operator is continuous and coersive when $f = \theta = g = 0$.

3. Given a hyperbolic nonlinear scalar conservation law has the form,

$$u(x,t)_t + f(u(x,t))_x = 0$$

(a) Use von Neumann analysis when $f(u) = \alpha u, \alpha \; real$, to determine that the finite difference scheme based on forward difference in time and centered difference in space is L_2 unstable if $\Delta t = \lambda \Delta x$.
(a) Present the Lax-Friedrichs scheme and show that it is consistent and on conservation form.
(c) Define an upwind scheme if $f(u) = Au$ where A is a matrix with real distinct eigenvalues