Algebraic Topology Prelim, August 2019

1. Let $X = T^2 \vee T^2$ be the wedge of two tori. Describe two connected 2-sheeted covering spaces X_1 and X_2 of X such that (the graded singular homology groups) $H_*(X_1)$ and $H_*(X_2)$ are not isomorphic. Compute $H_*(X_1)$ and $H_*(X_2)$.

2. A group G is *finitely presentable* if it has a presentation with a finite number of generators and relators: $\langle x_1, ..., x_m : r_1, ..., r_n \rangle$. The *deficiency* $\mathrm{def}(G)$ of G is defined to be the maximum of $(m - n)$ over all finite presentations of G. Let G be finitely presentable and let H be a subgroup of G of finite index k. Show that H is finitely presentable and $\mathrm{def}(H) \geq k(\mathrm{def}(G) - 1) + 1$. State carefully any theorems that you use.

3. Show that there is no retraction from $S^4 \times D^3$ to $S^4 \times S^2$.