PRELIMINARY EXAMINATION IN ANALYSIS

Part II, Complex Analysis

January 10, 2017

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1. Determine all bijective conformal self-maps of $\mathbb{C} \setminus \{0, 1\}$.

2. Let Ω be a non-empty open subset of $\mathbb{C}$, and let f be a continuous function on Ω. Suppose that $f_1, f_2, f_3, \ldots$ are analytic on Ω, and that

$$\lim_{n\to\infty} \int_D \left| f_n(x+iy) - f(x+it) \right| dxdy = 0 ,$$

for every closed disk $D \subset \Omega$. Show that f is analytic, and that $f_n \to f$ uniformly on compact subsets of Ω.

3. Prove that the range of the entire function $z \mapsto z^2 + \cos(z)$ is all of $\mathbb{C}$.

4. Determine the partial fraction expansion for $z \mapsto \dfrac{1}{z \sin z}$.