Preliminary exam: Numerical Analysis, Part A, January 6, 2017

Name ____ , EID _______________

1. Given a system of linear equations $Ax = b$ with A strictly positive definite.
(a) Show that the system is nonsingular.
(b) Define its Cholesky decomposition and determine the number of multiplications needed in the solution..
(c) Give the pseudo inverse of A and show how it can be used to solve over and under determined systems. Discuss properties of the solutions.

2. (a) Define Newton's method for minimization of a function $f(x), x \subset R^d$ which has a unique minimum at x_0.
(b) Prove that the method converges if $d = 1$ and,

$$f(x) \subset C^3(R),\ f''(x_0) \geq \delta > 0,\ f'''(x) > 0 \ \mathit{for}\ x > x_0,\ f'''(x) < 0 \ \mathit{for}\ x < x_0.$$

(c) If the minimization is constrained such that $x \in \Omega \subset R^d$ describe a penalty method for the minimization of the constrained problem.

3. The midpoint rule for numerical integration with error term is,

$$\int_{-1}^{1} f(x)\,dx = 2f(0) + \frac{1}{3} f''(\xi).$$

(a) Show that the midpoint rule is a Gauss quadrature method.
(b) Derive an asymptotic expansion in the step size parameter h for the composite midpoint rule.
(c) Divide the interval (-1,1) into 2n subintervals with $h = 1/n$ for the composite midpoint rule. Estimate the number of intervals needed to have an accuracy of 10^{-4} when,

$$f(x) = \exp(2x)\sin(x).$$