Preliminary exam: Numerical Analysis, Part B, January 6, 2017

Name:____ EID:________________

1. Consider the ordinary differential equation initial value problem,

$$x'(t) = f(x(t)), \quad t > 0,$$
$$x(0) = x_0$$

(a) For $f = -ax + g(x)$ what value of λ should be used in the model equation $x' = \lambda x$ for stiff problems to monitor the step size.
(b) To have a decaying solution, $|x(t_n)| \to 0, as\ t_n \to \infty,$ for the Euler approximation what conditions on $a, g, \Delta t$ are sufficient?
(c) Give a second order, two-stage, explicit Runge-Kutta method for the above ODE. Can such a method be A-stable?

2. The following elliptic PDE is given,

$$-\nabla \cdot a(x,y)\nabla u + b \cdot \nabla u + cu = f(x,y), \quad 0 < x < 1,\ 0 < y < 1,\ 0 < a \le a(x,y) \le A$$
$$u = d_1(x,y),\ x = 0\ and\ x = 1, 0 < y < 1,$$
$$u_y = d_0 u + d_2(x,y), y = 0\ and\ y = 1, 0 < x < 1,$$

(a) Rewrite the equation on weak form.
(b) Show that the relevant bilinear form is continuous and coersive and that the relevant linear form is continuous when $d_0 = d_1 = d_2 = 0$ for appropriate values of the vector *b* and constant *c* > 0. Give the fundamental error estimate for a finite element approximation based on the weak form in terms of the best approximation in the space of basis function.
(c) Prove coersivity if the conditions in (b) are valued but when also $b = 0, c = 0$.

3. Consider the initial value problem,

$$u_t + u^2 u_x = \varepsilon u_{xx}$$

for $\varepsilon > 0, -1 < x < 1, t > 0,$ with smooth initial data and periodic boundary condition.
(a) Construct and explicit converging finite difference or finite volume method for the problem above.
(b) What extra conditions are needed to guarantee convergence as $\varepsilon \to 0$?
(c) Determine the $\Delta t / \Delta x^2$ ratio for stability for the Euler-centered difference approximation as $u \to 0$?