

Algebra Prelim part A

August 16, 2016

Directions: You have 90 minutes. Solve three of the four problems. Clearly mark which ones you want graded.

- A1.** For K a field, show that $K[X]$ is a PID and $K[X, Y]$ is not a PID.
- A2.** Show that if G is a nonabelian finite group, then the center of G has index at least 4. Show that equality can occur.
- A3.** Show that a group of order $2004 = 2^2 \cdot 3 \cdot 167$ must be solvable. Give an example of a group of order 2004 in which a Sylow 3-subgroup is not a normal subgroup.
- A4.** Let R be a (possibly noncommutative) ring which contains a field K , and assume R is finite dimensional as a vector space over K . If R has no zero divisors, show that R is a division ring.