Algebra Prelim part B

Auguest 16, 2016

Directions: You have 90 minutes.

B1. Prove that $\mathbf{Q}(\sqrt{2})$ is not isomorphic to $\mathbf{Q}(\sqrt{3})$.

B2. Prove that $\mathbf{Q}(\sqrt{2+\sqrt{2}})$ is Galois over $\mathbf{Q}$, and find its Galois group.

B3. Let F be a field contained in the ring of $n \times n$ matrices over $\mathbf{Q}$. Prove that $[F : \mathbf{Q}] \leq n$.