Algebra Prelim Part A January 2015

1. Show that a group G of order pqr, for primes $p < q < r$, has a normal cyclic subgroup H. Show, furthermore, that H can be chosen such that G/H is also cyclic.

2. Let R be a PID with field of fractions F, let S be a subring of F which contains R, and let A be an ideal of S. Prove that $A \cap R$ is an ideal of R. If $A \cap R = Rd$, prove that $A = Sd$.

3. Let p be a prime and let S be a set of cardinality a power of p. Suppose G is a finite group that acts transitively on S (so if $s, t \in S$, then there exists $g \in G$ such that $gs = t$) and let P be a Sylow p-subgroup of G. Prove that P acts transitively on S.