Numerical Analysis Prelim, Part I (Fall Material)

August 19, 2015

Name: ________________________________ EID: ________________________________

1. Consider a linear system of equations, $Ax = b$, and a related iterative algorithm, $x^{n+1} = Bx^n + f$, $n = 0, 1, \cdots$.

(a) Prove convergence under appropriate sharp conditions on the matrix B, the vector f and the eigenvalues of B.

(b) Show that the Jacobi method satisfies these conditions if A is strictly diagonally dominant.

(c) Show that the Jacobi method converges in a finite number of iterations if A is upper triangular.

2. Consider the optimization problem: $\min_{x \in \Omega} f(x)$

(a) Define Newtons method for this problem when $\Omega = R^n$ and prove quadratic convergence under appropriate conditions on f if $\Omega = R^1$.

(b) Formulate the Kuhn-Tucker or similar conditions for the case $\Omega = \{x \in R^2, |x| \leq 1\}$.

(c) Show how the optimization problem on the bounded domain can be transformed into an unconstrained problem by adding a penalty function.

3. (a) Prove that interpolation of n points by polynomials of degree $n - 1$ has a unique solution.

(b) Show that interpolation of n points by a linear combination of n monomials (form: x^m) of different degrees may not exist or be unique.

(c) Show that under certain conditions Fourier interpolation (basis functions e^{inx}, $n = 0, 1, \cdots, N$) has a unique solution. Give an example when Fourier interpolation is not unique.