# PRELIMINARY EXAM IN ANALYSIS
# PART I – REAL ANALYSIS.

AUGUST 25, 2014 – TIME: 1:00–2:30 PM

Name (**print**): ____________________________________________ UT ID: _____________

**Please write clearly, and staple your work with the signed exam sheet on top !**

## Problem 1

Let $1 \leq p, q \leq \infty$ with $\frac{1}{p} + \frac{1}{q} = 1$. Show that if $f \in L^p(\mathbb{R}^n)$ and $g \in L^q(\mathbb{R}^n)$ then $f * g$ is bounded and continuous on $\mathbb{R}^n$.

## Problem 2

Let $f \in L^1(X, \mu)$. Prove that for every $\epsilon > 0$, there exists $\delta > 0$ such that

$$\Big| \int_A f d\mu \Big| < \epsilon$$

holds whenever $A$ is a measurable subset of $X$ with $\mu(A) < \delta$.

## Problem 3

Let $p \in [1, \infty)$ and suppose $\{f_n\}_{n=1}^{\infty} \subset L^p(\mathbb{R})$ is a sequence that converges to 0 in the $L^p$ norm. Prove that one can find a subsequence $\{f_{n_k}\}$ such that $f_{n_k} \to 0$ almost everywhere.

## Problem 4

Recall that a sequence $\{f_i\}_{i=1}^{\infty}$ of real-valued measurable functions on the real line is said to *converge in measure* to a function $f$ if

$$\lim_{i \to \infty} \lambda(\{x \in \mathbb{R} : \ |f_i(x) - f(x)| \geq \epsilon\}) = 0 \quad \forall \epsilon > 0$$

where $\lambda$ denotes Lebesgue measure on $\mathbb{R}$. Suppose that in addition to this, there exists an integrable function $g$ such that $|f_i| \leq g$ for all $i$. Prove that $\{f_i\}_{i=1}^{\infty}$ converges to $f$ in $L^1(\mathbb{R})$.

## Problem 5

Show that, if $f \in L^4(\mathbb{R})$ then

$$\int \big| f(\lambda x) - f(x) \big|^4 dx \to 0$$

as $\lambda \to 1$.