PRELIMINARY EXAM IN ANALYSIS
PART II – COMPLEX ANALYSIS.

AUGUST 25, 2014 – TIME: 2:40–4:10 PM

Name (**print**): ____ UT ID: ____________

Please write clearly, and staple your work with the signed exam sheet on top !

Problem 1

Find a function f such that f is holomorphic in $\mathbb{C} \setminus \mathbb{N}$, and that at each positive integer n, f has a pole of order n.

Problem 2

Suppose that f is a meromorphic function on the punctured disk $0 < |z| < 1$, having poles at $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$. Show that for every $r > 0$, the restriction of f to $0 < |z| < r$ has a range that is dense in $\mathbb{C}$.

Problem 3

Suppose $\Omega \subset \mathbb{C}$ is bounded, $\{f_n\}$ is a sequence of continuous functions on $\overline{\Omega}$ that are holomorphic in Ω and $\{f_n\}$ converges uniformly on the boundary of Ω. Prove that $\{f_n\}$ converges uniformly on $\overline{\Omega}$.

Problem 4

Assume that f and g are holomorphic functions in an open and connected region Ω such that $|f(z)|+|g(z)|$ is constant for all $z \in \Omega$. Prove that then, f and g must both be constant.

Problem 5

Let $(f_1, f_2, \ldots)$ be a sequence of analytic functions on a connected open domain Ω, converging to a non-constant function f, uniformly on compact subsets of Ω. Show that if each f_n is one-to-one, then so is f.