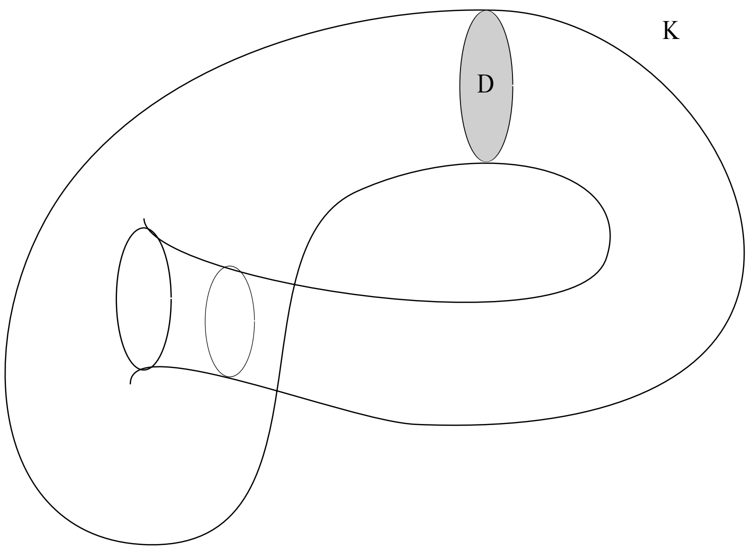
Preliminary Examination in Topology: August 2014

Algebraic Topology portion

Instructions: Do all questions.

Time Limit: 90 minutes.

1. Let X be the 2-complex obtained from a Klein bottle K by attaching a 2-cell D along an essential orientation-preserving curve. (The figure below shows the image of X under the usual map to $\mathbb{R}^3$.)



a. Compute the first and second homology groups of X using a Mayer-Vietoris sequence.
b. Compute the fundamental group of X.
c. Classify all connected covering spaces of X. (Describe the spaces clearly.)
d. Compute the homology groups of the finite covers you found in part **c**. (Hint: Euler characteristic).
e. Is there a retraction of X onto K? (Describe or prove nonexistence.)
f. Is there an embedded loop in X onto which X retracts? (Describe or prove nonexistence.)

2. Prove the homotopy lifting theorem: *Let* $p : \tilde{X} \to X$ *be a covering. Let* $F : [0,1] \times [0,1] \to X$ *be a map such that* $F(0,t) = F(1,t) = x$ *for* $0 \leq t \leq 1$ *and for some* $x \in X$. *Let* $\tilde{x} \in p^{-1}(x)$. *There is a map* $\tilde{F} : [0,1] \times [0,1] \to \tilde{X}$ *which satisfies* $p \circ \tilde{F} = F$ *and* $\tilde{F}(0,t) = \tilde{x}$ *when* $0 \leq t \leq 1$.