Algebra I Prelim Exam

August 20, 2025

The exam is out of 30. The passing cutoff is 20 points.

Question 1 (10 points). *Let G be a group of order $504 = 2^3 3^2 7$.*

1. *Prove that G is not isomorphic to a subgroup of the alternating group A_7.*

2. *Assume that G is simple, then determine the number of 3-Sylow subgroups of G.*

Question 2 (5 points). *Show that for G a finite abelian group, any irreducible n-dimensional complex representation is one-dimensional.*

Question 3 (10 points). *Let R be an integral domain such that every prime ideal of R is principal.*

1. *Consider the set of ideals of R which are not principal. Prove that this set is non-empty, then it contains an element I which is maximal under inclusion.*

2. *Prove that R is a principal ideal domain. (Hint: Show that I is principal.)*

Question 4 (5 points). *Prove that the rings $F[x,y]/(y^2-x)$ and $F[x,y]/(x^2-y^2)$ are non-isomorphic for any field F.*