Prelim, Numerical Analysis I (fall course), 12:00-2:00 PM, August 21, 2026

1. Consider the matrix $A = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 6 & 1 \\ 0 & 1 & 6 \end{pmatrix}$. (a) Define the Jacobi iterative method for the equation, $Ax = b$, and prove that it will converge.
(b) Use Gershgorin or other arguments to define a small domain containing the eigenvalues of A and do the same for the singular values of A.
(c) Propose an inverse iteration method for approximation of the smallest real eigenvalue of A.

2. Consider the constrained minimization problem $\min_{x_1 \geq 0} (x_1 + x_1^4 + x_2 + x_2^2)$.
(a) Define Newton's method for minimization without constraint and for the constrained problem using a penalty technique.
(b) Formulate Newton's method for the problem when (x_1, x_2) is restricted to the constraint boundary, $x_1 = 0$, and
(c) Show that it then (when $x_1 = 0$) converges in a finite number of steps. Will the unconstrained iterations in (a) also converge in a finite number of steps?

3. A one-sided quadrature formula for $\int_0^h f(x)dx$ has the form a$hf(0) + bh^2 f'(0)$. Assume $f \in C^\infty(R)$.
(a) Determine a, b such that the error is of order $O(h^3)$
(b) Prove that the error satisfies an asymptotic expansion in powers of h.
(c) Discuss how an asymptotic expansion can be used to derive Richardson extrapolation.