ALGEBRA PRELIMINARY EXAM: PART I

(1) (a) Let G be a group and A an abelian group. Define a structure of group on $\text{Hom}(G, A)$. Is it abelian?
   (b) Prove or disprove: $\text{Hom}(\mathbb{Z}/9\mathbb{Z}, \mathbb{Z}/3\mathbb{Z})$ and $\text{Hom}(\mathbb{Z}/3\mathbb{Z} \times \mathbb{Z}/3\mathbb{Z}, \mathbb{Z}/3\mathbb{Z})$ are isomorphic groups.

(2) (a) Prove that a finite integral domain is a field.
   (b) Let R be an integral domain with unit element such that $R[x]$ is a principle ideal domain. Prove that R is a field.

(3) Show that if the free R-modules R^n and R^m over a non-zero commutative ring R are isomorphic, then $n = m$. Is this true if R is non-commutative?